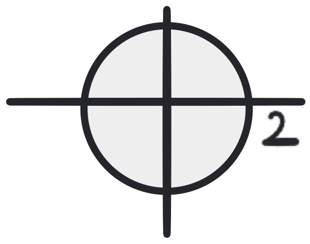
Quiz 19- 16.7ii

Evaluate the flux integral $\iint_S \vec{F} \cdot d\vec{S}$ where $\vec{F}(x,y,z) = \langle -x, -y, z^3 \rangle$ and S is the part of the paraboloid $z = 4 - x^2 - y^2$ above z=0.

$$z = 4 - x^2 - y^2 = g(x,y)$$



$$\underbrace{z - 4 + x^2 + y^2}_{G(x,y,z)} = 0$$

$$\vec{\nabla} G = \langle 2x, 2y, 1 \rangle \text{ up} \checkmark$$

$$\vec{F} = \langle -x, -y, (4 - x^2 - y^2)^3 \rangle$$

$$\vec{F} \cdot \vec{\nabla} G = -2x^2 - 2y^2 + (4 - x^2 - y^2)^3$$

$$\iint_S \vec{F} \cdot d\vec{S} = \iint_D -2(x^2+y^2) + (4 - (x^2+y^2))^3 \, dA$$

$$= \int_0^{2\pi}\int_0^2 -2r^3 + (4-r^2)^3 r \, dr\, d\theta$$

$$\int_0^{2\pi}\int_0^2 -2r^3 \, dr\, d\theta + \int_0^{2\pi}\int_0^2 (4-r^2)^3 r \, dr\, d\theta$$

$$u = 4 - r^2$$
$$du = -2r\,dr$$

$$\int_0^{2\pi} -\frac{1}{2} r^4 \Big]_0^2 \, d\theta + -\frac{1}{2}\int_0^{2\pi}\int_4^0 u^3 \, du\, d\theta$$

$$-16\pi - \frac{1}{2}\int_0^{2\pi} \frac{u^4}{4}\Big]_4^0$$

$$-16\pi - \frac{1}{2} 2\pi (0 - 64)$$

$$-16\pi + 64\pi = 48\pi$$